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Problem-Based Learning for Teaching Place Value and Basic Arithmetic Operations to Second-Grade Elementary Students

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Abstract:
Early mathematics learning requires more than procedural fluency: children must understand what numbers mean and use operations in meaningful situations. This study describes the implementation of Problem-Based Learning (PBL) in teaching place value and basic arithmetic operations to second-grade students and examines changes in their understanding across three lessons. Fourteen students from one class participated in three 2 × 35-minute lessons conducted from 28 to 30 September 2026. The lessons followed the five phases of PBL and incorporated locally available concrete objects, number cards, and student worksheets. Data were collected through pretests and posttests, lesson-by-lesson assessments, observations, worksheets, exercises, and teacher notes, and were analyzed descriptively. Student achievement was assessed using test scores, while understanding categories were determined based on the assessment criteria applied in the study. The mean score increased from 64.00 to 83.86 (mean gain = 19.86), with all 14 students showing improvement (Wilcoxon signed-rank $p < .001$; mean normalized gain = 0.57, classified as medium). The proportion of students categorized as demonstrating understanding increased from 71.43% to 85.71% and 92.86% across the three lessons. Nevertheless, because this study employed a single-group design without a comparison group, the observed improvements cannot be attributed exclusively to PBL. The findings indicate positive changes in students' understanding during the implementation and suggest that contextual problems, concrete representations, and guided scaffolding may support early number understanding.

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Introduction (مقدمة)

Early mathematics learning involves more than carrying out calculation procedures; children must develop an understanding of numbers, recognize relationships between quantities, and use mathematical operations in meaningful situations. These abilities are closely related to number sense, which provides a foundation for understanding numerical relationships and performing arithmetic operations. Place value is particularly important because it helps children understand how the position of a digit determines its value and supports the development of more advanced skills, such as addition and subtraction with regrouping. Children who do not develop a sound understanding of place value may experience persistent difficulties in subsequent mathematics learning (Mix, Prather, Smith, & Stockton, 2014). Research involving second- and third-grade students has shown that place value understanding is associated with their ability to represent multi-digit numbers correctly (Herzog & Fritz, 2022). Similarly, Amelia and Herman (2024) reported that 75% of 20 second-grade students achieved the minimum passing score on a place value assessment, with a mean score of 70. These findings indicate that place value remains a challenging concept for some students at the early elementary level. Furthermore, conceptual and procedural knowledge develop in a mutually supportive relationship, emphasizing the importance of instruction that connects mathematical symbols and procedures with their underlying meanings (Rittle-Johnson, 2017).

The classroom context of the present study reflects these challenges. Based on the teacher's observations and students' classroom work, the students were generally able to recall and order numbers but experienced difficulties in identifying the value of a digit according to its position and applying basic arithmetic operations to everyday problems. The school also had limited learning facilities and internet access, making technology-dependent instruction less practical. These conditions highlight the need for instructional approaches that are concrete, accessible, contextual, and interactive. One approach is the use of concrete manipulatives, which can help students connect physical quantities with numerical symbols and abstract mathematical concepts. A meta-analysis of 55 studies involving 7,237 participants found that mathematics instruction using concrete manipulatives generally produced small to moderate learning benefits compared with instruction relying solely on abstract symbols (Carbonneau, Marley, & Selig, 2013). However, the effectiveness of manipulatives depends partly on how they are integrated into instruction. Carbonneau and Marley (2015) also found that the level of instructional guidance and the characteristics of manipulatives influenced preschool children's mathematics learning. Thus, concrete materials need to be accompanied by meaningful learning activities and appropriate teacher guidance.

Problem-Based Learning (PBL) provides an instructional framework through which contextual problems, concrete materials, and teacher guidance can be integrated. PBL places problems at the beginning of the learning process and encourages students to identify problems, gather relevant information, investigate possible solutions, present their findings, and evaluate their learning process, while the teacher facilitates and guides students' activities (Arends, 2012; Rusman, 2017; Yew & Goh, 2016). In primary mathematics education, PBL has been associated with improvements in students' mathematical abilities. A meta-analysis of 16 primary studies reported a positive overall effect of PBL on primary students' mathematical abilities compared with conventional instruction (Suparman, Tamur, Yunita, Wijaya, & Syaharuddin, 2021). Similarly, Fitriyah, Putro, and Apino (2022) identified a positive overall effect of PBL on students' mathematical reasoning based on a meta-analysis of Indonesian studies. More recently, a systematic review of PBL in K–12 mathematics education identified a variety of instructional

models and learning outcomes across different mathematical domains. However, the review found that geometry and algebra were more frequently addressed than arithmetic, indicating that arithmetic remains a comparatively less represented topic in the reviewed PBL literature (Ayari et al., 2025). These findings support the relevance of PBL in mathematics education while also highlighting the need to examine its application to foundational mathematical concepts at the lower elementary level.

Despite the growing body of research on PBL in mathematics education, several aspects require further investigation. Existing studies and research syntheses have frequently emphasized broad mathematical abilities, problem-solving skills, and reasoning outcomes across different educational levels. Comparatively less attention has been given to how PBL can be implemented specifically to teach place value and basic arithmetic operations in a second-grade classroom, particularly when learning resources are limited. Furthermore, there is a need for a more detailed account of how students' understanding develops across successive lessons rather than being examined only through overall learning outcomes. Another important consideration is the role of instructional guidance. Although PBL encourages active student participation and independent problem-solving, young learners may require structured support to understand problems, select appropriate representations, and connect concrete experiences with mathematical concepts. Research on guided inquiry and scaffolding emphasizes the importance of providing appropriate guidance during problem-centered learning activities (Kirschner, Sweller, & Clark, 2006; Belland, Walker, & Kim, 2017; Lazonder & Harmsen, 2016). Therefore, examining PBL alongside contextual problems, concrete manipulatives, and guided scaffolding is relevant for understanding how foundational mathematical concepts can be taught in an accessible and meaningful way.

The novelty of this study lies in documenting the integration of three complementary instructional elements within a single PBL sequence for second-grade mathematics learning. First, contextual problems connect place value and basic arithmetic operations with situations that are familiar to students. Second, locally available concrete materials, number cards, and student worksheets provide opportunities for students to represent and explore numerical concepts through hands-on activities. Third, guided scaffolding enables the teacher to provide structured assistance throughout the PBL phases, helping students move from concrete representations toward mathematical symbols and procedures. Rather than claiming that these elements constitute entirely new teaching strategies, this study highlights their combined application in a low-resource second-grade classroom and examines students' understanding across three consecutive lessons.

Accordingly, this study aims to describe the implementation of PBL in teaching place value and basic arithmetic operations to second-grade elementary students, examine changes in students' understanding across three lessons and between pretest and posttest results, and identify factors that supported or constrained the implementation. Specifically, the study addresses the following research questions:

1. How is Problem-Based Learning implemented in teaching place value and basic arithmetic operations to second-grade elementary students?
2. How does students' understanding of place value and basic arithmetic operations develop across the three lessons and between the pretest and posttest?
3. What factors support or constrain the implementation of Problem-Based Learning in the classroom?

This study adopts a descriptive single-group design and does not aim to establish a causal relationship between PBL implementation and students' learning outcomes. Instead, it provides a contextual account of how a concrete and guided PBL sequence can be implemented to support early number understanding in a second-grade elementary classroom.

Method (منهج)

Research Design

This study used a descriptive quantitative approach with a one-group pretest–posttest design, supplemented by records of students' understanding at the end of each lesson. The design is pre-experimental: there was no comparison group and no random assignment (Creswell & Creswell, 2018). Its purpose was to describe the implementation of PBL and the development of students' understanding within one class, not to estimate a causal effect.

Participants and Setting

The participants in this study were 14 second-grade students from one elementary school in Indonesia. The students were enrolled in Phase A of the Indonesian curriculum. All 14 students participated in the three learning sessions and completed both the pretest and posttest. To maintain confidentiality, each student was assigned an identification code (S1–S14). The study was conducted from 28 to 30 September 2026, with each learning session lasting 2 × 35 minutes. The school had limited learning facilities and internet access, and digital devices were not available for classroom activities. Therefore, the learning activities relied on locally available concrete materials, number cards, pictures, and student worksheets (LKPD).

Learning Procedure

The learning activities were conducted over three consecutive sessions from 28 to 30 September 2026. Each session lasted 2 × 35 minutes and followed the five phases of Problem-Based Learning (PBL): (1) orienting students to the problem, (2) organizing students for learning, (3) guiding individual or group investigation, (4) developing and presenting the results, and (5) analyzing and evaluating the problem-solving process. The learning activities, instructional materials, and implementation of the five PBL phases in each session are presented in Table 1.

Table 1. Learning Activities Based on the Five PBL Phases

Session	PBL Phase	Learning Activities	Instructional Materials
1. Place Value (28 September 2026)	1. Orienting students to the problem	Students were introduced to problems involving the representation of two-digit numbers and the value of their digits.	Pictures and number cards
	2. Organizing students for learning	Students were organized into small groups and given tasks related to representing two-digit numbers.	Number cards and LKPD
	3. Guiding individual or	Students represented the numbers 23, 35, 42, and 51	Stones, sticks, leaves, seeds,

	group investigation	using concrete objects, grouped the objects into tens and ones, and identified the value of each digit.	and number cards
	4. Developing and presenting results	Students explained their representations of tens and ones and presented their answers.	Concrete objects and number cards
	5. Analyzing and evaluating the problem-solving process	Students reviewed their answers and discussed the relationship between digit position and digit value.	LKPD and teacher-guided discussion
2. Addition and Subtraction (29 September 2026)	1. Orienting students to the problem	Students were introduced to contextual problems involving addition and subtraction.	Pictures and contextual problems
	2. Organizing students for learning	Students worked in small groups to identify the information needed to solve the problems.	LKPD
	3. Guiding individual or group investigation	Students identified known information, selected the appropriate arithmetic operation, and calculated using concrete objects.	Stones, sticks, leaves, seeds, and number cards
	4. Developing and presenting results	Students explained the operations they selected and presented their solutions.	Concrete objects and LKPD
	5. Analyzing and evaluating the problem-solving process	Students reviewed their problem-solving steps and discussed whether their answers were appropriate.	LKPD and teacher-guided discussion
3. Integration of Place Value and Basic Arithmetic Operations (30 September 2026)	1. Orienting students to the problem	Students were introduced to a multi-step contextual problem involving 23 stones, followed by adding 8 stones and removing 6 stones.	Contextual problems and pictures
	2. Organizing students for learning	Students worked in small groups to understand the problem and plan how to solve it.	LKPD and number cards
	3. Guiding individual or group investigation	Students represented the quantities, applied place-value concepts, performed the required operations, and checked their calculations.	Stones, sticks, leaves, seeds, and number cards
	4. Developing and presenting results	Students communicated their solutions and explained the strategies used to solve the problem.	Concrete objects and LKPD

	5. Analyzing and evaluating the problem-solving process	Students reviewed their strategies, checked their answers, and reflected on the problem-solving process.	LKPD and teacher-guided discussion

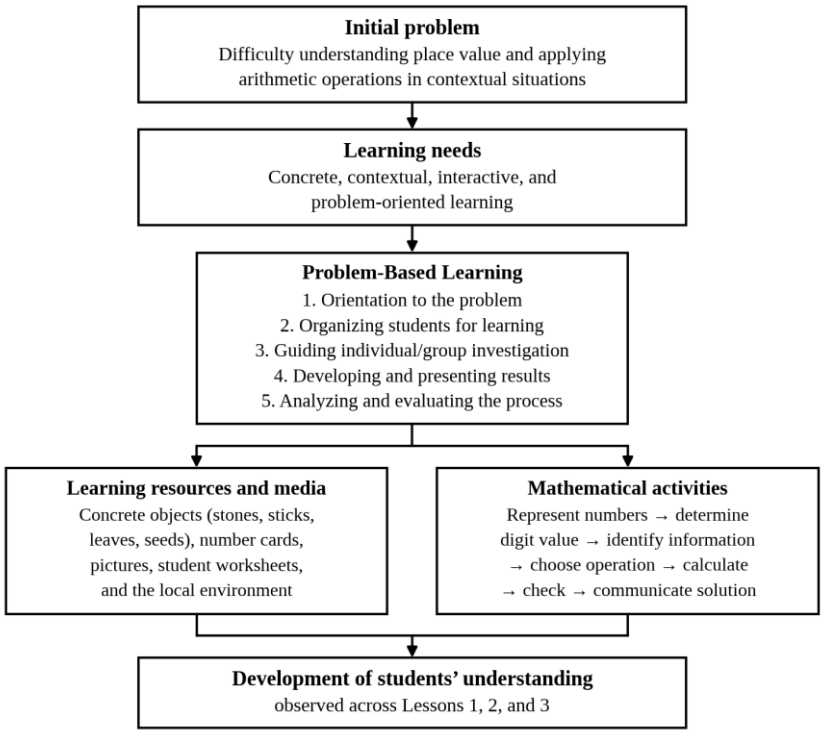

Figure 1. Conceptual framework of the PBL implementation

Instruments and Data Collection

The study used several instruments to collect data on students' mathematical understanding and learning activities. These instruments included a pretest, a posttest, individual exercises, student worksheets (LKPD), observation sheets, teacher notes, reflection records, and activity documentation.

The pretest was administered before the first learning session, while the posttest was administered after the third session. Both tests were designed based on four learning indicators: (1) recognizing the value of a digit, (2) determining the value of a digit based on its position, (3) performing addition and subtraction, and (4) applying these concepts to solve mathematical problems. Each student's test performance was recorded individually, with scores expressed on a scale of 0–100. In addition to the pretest and posttest, an individual five-item exercise was administered at the end of each learning session to assess students' understanding of the material covered in that session. Student worksheets (LKPD) were also used to collect evidence of students' responses and problem-solving activities during the lessons.

Observational data were collected throughout the three learning sessions using observation sheets and teacher notes. These records were used to document students' participation, responses to learning activities, and progress in understanding the mathematical concepts. Reflection records and activity documentation provided additional information about the implementation of the learning activities. At the end of each session, students' understanding was also recorded using two descriptive categories: "understanding" and "not yet understanding."

The quantitative data from the pretest, posttest, and individual exercises were used to describe students' learning outcomes and their development across the three sessions. Meanwhile, the observation sheets, teacher notes, reflection records, and activity documentation were used as supporting evidence to describe the learning process. These sources were considered complementary and were not treated as independent measures of the causal effect of PBL.

Data Analysis

Data were analyzed descriptively. For the pretest and posttest, the minimum, maximum, mean, median, and standard deviation were computed, together with each student's gain ($\Delta X = X_{post} - X_{pre}$). The proportion of students in a category was computed as $P = f/N \times 100\%$, where f is the number of students in the category and N is the total number of students. Relative improvement was expressed as normalized gain, $g = (X_{post} - X_{pre})/(X_{max} - X_{pre})$, where X_{max} is the maximum possible score, assumed here to be 100; g was interpreted as high (≥ 0.70), medium (0.30–0.69), or low (< 0.30) (Hake, 1998). Because of the small sample and bounded scores, the pretest–posttest difference was also checked with a Wilcoxon signed-rank test (Field, 2018) at $\alpha = .05$, and the mean gain is reported with a 95% confidence interval. These procedures are supplementary: the analysis remains descriptive and does not support causal claims.

Data Quality and Ethics

Data quality was supported by using several complementary sources (tests, exercises, worksheets, observation, teacher notes, and reflections) and by comparing the quantitative results with process evidence. The module planned formative assessment in every lesson through observation, worksheets, a five-item exercise, and teacher notes; such assessment is intended to give evidence of learning that teachers can act on (Black & Wiliam, 2018). Student names were replaced with codes (S1–S14), only information needed for the analysis is reported, and photographs of students are not included. Because there was no comparison group, the findings are interpreted for this class only.

Result (نتائج)

Implementation of PBL

All five PBL phases were carried out in each of the three lessons. In Lesson 1, the problem was linked to objects students could find around them, and groups used objects, number cards, and worksheets to represent 23, 35, 42, and 51 as tens and ones; the teacher's guiding questions directed attention to the value of each digit. In Lesson 2, students first identified what was known in an addition or subtraction problem, then chose an operation, calculated with objects, and explained their solution to the class. In Lesson 3, students solved multi-step problems, checked their answers, and communicated their solution strategies. Table 2 summarizes how the syntax was implemented, the adjustments made, the constraints encountered, and the responses to them. The adjustments changed the learning resources and the amount of support, not the sequence of the model's phases.

Table 2. Implementation of PBL, adjustments, constraints, and responses

Aspect	Description
PBL syntax	Orientation to the problem, organization of students, individual/group investigation, presentation of results, and analysis and evaluation were applied in all three lessons.
Adjustments	Concrete objects and materials available around the school (stones, sticks, leaves, and seeds) were used, and tasks were adapted to second-grade students.
Constraints	Limited facilities and limited internet access; some students still needed help to understand the problem and to choose the operation.
Responses	Using the surrounding environment as a concrete learning resource, guiding questions, scaffolding, and individual assistance for students who still had difficulty.

Pretest and Posttest Results

Table 3 presents the pretest and posttest descriptive statistics. The mean score rose from 64.00 (*SD* = 9.28) to 83.86 (*SD* = 7.12), a mean gain of 19.86 points (95% CI [17.62, 22.10]). The minimum score increased from 48 to 72, the maximum from 80 to 96, and the median from 64 to 84. All 14 students scored higher on the posttest. Individual gains ranged from 12 points (S8) to 24 points (S1, S2, S3, S10, and S11), as shown in Figure 2a. A Wilcoxon signed-rank test indicated that posttest scores were higher than pretest scores ($W = 0$, $p < .001$). The mean normalized gain was 0.57 (*SD* = 0.12; median = 0.55; range = 0.43–0.80); twelve students were in the medium category and two (S11 and S14) were in the high category.

Table 3. Descriptive statistics of pretest and posttest scores (n = 14)

Statistic	Pretest	Posttest
Minimum	48	72
Maximum	80	96
Mean	64.00	83.86
Median	64	84
Standard deviation	9.28	7.12

Students' Understanding in Each Lesson

Table 4 and Figure 2b show that the number of students categorized as understanding increased from 10 of 14 (71.43%) in Lesson 1 to 12 (85.71%) in Lesson 2 and 13 (92.86%) in Lesson 3, while the number not yet understanding fell from four to two and then to one. This corresponds to an increase of 14.28 percentage points between Lessons 1 and 2 and 21.43 percentage points between Lessons 1 and 3. One student remained in the not-yet-understanding category at the end of Lesson 3.

Table 4. Students' understanding in each lesson (n = 14)

Lesson	Date	Understanding	Not yet understanding	Understanding (%)
1	28 September 2026	10	4	71.43
2	29 September 2026	12	2	85.71
3	30 September 2026	13	1	92.86

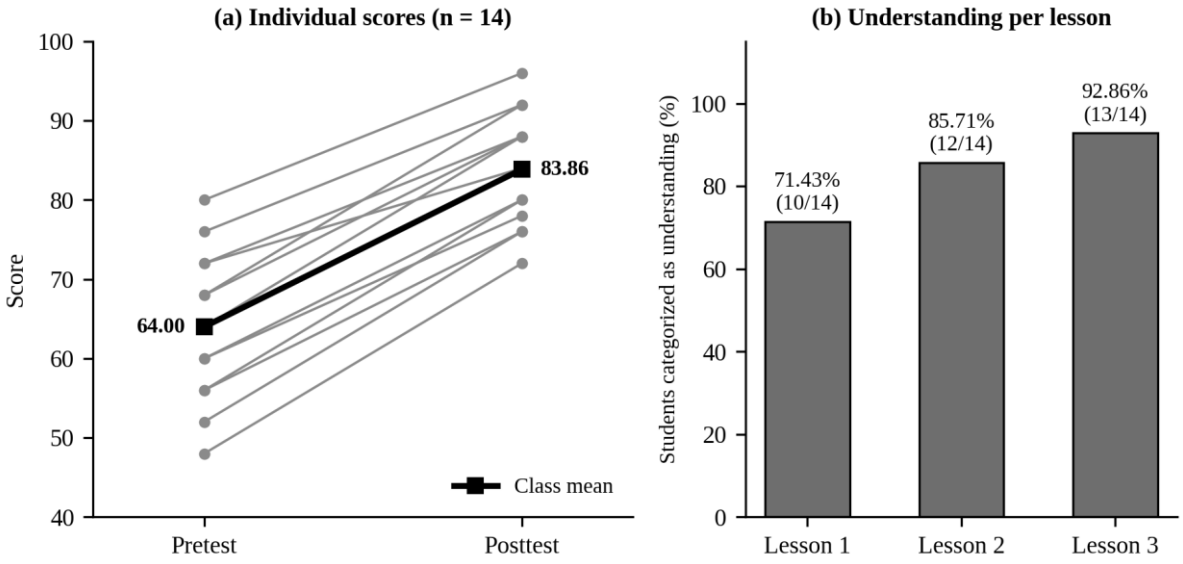

Figure 2. Pretest and posttest scores of individual students (a) and percentage of students categorized as understanding in each lesson (b)

Discussion (مناقشة)

This study described how PBL was implemented in three second-grade lessons on place value and basic arithmetic operations and how students' understanding developed. Three main findings emerged. First, the five PBL phases could be carried out in every lesson with low-cost, locally available materials. Second, all 14 students scored higher on the posttest, with a mean gain of 19.86 points. Third, the share of students categorized as understanding rose across the lessons from 71.43% to 92.86%, although one student had not yet reached understanding by the last lesson.

These results are consistent in direction with earlier work on PBL in primary mathematics. Andayani et al. (2019) also reported rising mastery across cycles, and syntheses report positive pooled effects of PBL on mathematical abilities and reasoning (Fitriyah et al., 2022; Suparman et al., 2021). The magnitudes, however, cannot be compared directly. Those studies differ in grade level, content, outcome construct, sample size, duration, and design, and many used comparison conditions or several cycles, whereas the present study followed one class through three lessons without a comparison group. The earlier work is therefore used here as a benchmark for the pattern of results, not for the size of the improvement.

The pattern across lessons mirrors the sequence of the module. Lesson 1 concentrated on the meaning of digit position, Lesson 2 on using operations in context, and Lesson 3 on combining both in multi-step problems, so the increase in understanding is consistent with a progression from concept to application. Several features of the implementation plausibly contributed, although the design cannot isolate their separate contributions. Problems gave students a reason to attend to place value and to choose an operation instead of only following a procedure. Concrete objects and number cards let students see, for example, that 23 is two groups of ten and three ones before working with symbols, which is the kind of link between representations and symbols that research on manipulatives emphasizes (Carbonneau et al., 2013). Group investigation and presentation required students to explain their reasoning, and the final check-and-communicate steps made the solution process visible to the teacher. These features fit the view that conceptual and procedural knowledge develop together (Rittle-Johnson, 2017).

Guidance played an important role. The teacher's guiding questions, the reuse of concrete objects, and individual help for students who struggled to understand the problem or to choose an operation were part of the design rather than additions to it. This agrees with the argument that minimally guided instruction is a weak approach (Kirschner et al., 2006) and with meta-analytic evidence that guidance and scaffolding support learning in inquiry and problem-centered settings (Belland et al., 2017; Lazonder & Harmsen, 2016). Belland et al. concerned computer-based scaffolds, and both meta-analyses cover many populations, so they are treated here as support for the principle of temporary, responsive assistance rather than as direct evidence for the present procedures.

Understanding did not develop at the same pace for all students. Four students were not yet categorized as understanding after Lesson 1, two after Lesson 2, and one after Lesson 3. The smallest score gain (12 points, S8) belonged to a student whose pretest score (72) was among the higher ones, leaving less room to improve; the student's normalized gain (0.43) was still in the medium category. These observations are in line with reports that place value is a persistent difficulty for some second graders (Amelia & Herman, 2024; Herzog & Fritz, 2022) and show that a problem-based sequence does not automatically bring every student to understanding at the same time. They also show why scaffolding and repeated opportunities to use concrete representations remain necessary even when the sequence is well organized.

The materials used (stones, sticks, leaves, and seeds) served two purposes: they made number representations visible and manipulable, and they set the problems in objects familiar to students. The local context was therefore treated as a source of low-cost material and familiar situations, not as cultural content. Because the school had limited facilities and internet access, the results suggest that a PBL sequence of this kind is feasible without digital technology. Similar progress in second graders' place value understanding has been reported for a designed teaching unit in action research in Colombia (Angulo Escamilla, Pulido Peñaloza, & Molano Rodríguez, 2017), which suggests that carefully structured units can help at this age in different settings.

Four implications follow for teaching in the lower grades. First, problems can be built from situations and objects students already know, so that place value and operations are learned for a purpose. Second, concrete objects can serve as a bridge from direct experience to number symbols and operations, provided that guiding questions link the objects to the mathematical idea. Third, scaffolding should target students who cannot yet identify relevant information or choose an operation. Fourth, checking and communicating the solution should remain part of the lesson so that students explain how they reached an answer, not only what the answer is.

Several limitations should be noted. The study involved one class of 14 students for three lessons and had no comparison group, so the gains may reflect other influences such as maturation, repeated testing, or teacher attention and cannot be attributed to PBL alone. The analysis was descriptive; the Wilcoxon test is a supplementary check on a small sample. The instruments were aligned with the learning indicators, but formal reliability and validity evidence is not reported, and the understanding categories rest on teacher judgment without a published scoring rubric. The posttest was given immediately after instruction, so retention is unknown. Future studies should use a comparison group, larger and more varied samples, validated instruments with explicit cut-off criteria, a longer implementation period, and a delayed posttest.

Conclusion (خاتمة)

This study described the implementation of Problem-Based Learning (PBL) in teaching place value and basic arithmetic operations to second-grade students and examined the

development of their understanding. The findings showed that the five PBL phases were implemented across three lessons using concrete, locally available materials, guiding questions, and scaffolding. Students' mean scores increased from 64.00 to 83.86, and all 14 students demonstrated improvement. Furthermore, the proportion of students categorized as understanding increased from 71.43% in the initial assessment to 85.71% and 92.86% across the subsequent lessons. These findings indicate positive developments in students' understanding during the PBL implementation within the observed class. However, the results should be interpreted within the scope of the 14 students and three lessons, as the single-class design and short implementation period do not allow causal conclusions or generalization to other student groups.

This study provides a documented example of a concrete, guided, and low-resource PBL sequence that teachers may adapt when teaching early number concepts. It also demonstrates how students' understanding can be monitored throughout successive lessons. Nevertheless, the study is limited by the absence of a comparison group, the involvement of only one class, and the short implementation period. Future research should involve larger and more diverse student groups, comparison groups, longer implementation periods, validated assessment instruments, and delayed posttests to examine the consistency and retention of students' understanding over time.

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